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Diffuse Reflectance of the Optically Deep Sea Under Combined Illumination of Its Surface

Vladimir I. Haltrin
Naval Research Laboratory, Ocean Sciences Branch, Code 7331, Stennis Space Center, MS 39529-5004, USA
Phone: 601-688-4528, fax: 601-688-5379, e-mail: <haltrin@nrlssc.navy.mil>

Abstract — The processing of remotely measured color imagery involves knowledge of the diffuse reflectance of the oceanic water. This paper presents two approaches to this problem. The first one is analytical and consists of an approximate solution to the radiative transfer equation. As a result it gives an equation for diffuse reflectance of the sea as a function of inherent optical properties, the sun elevation angle and the ratio of the direct illumination by sun to the diffuse illumination by sky. The second approach is a numerical Monte Carlo simulation. The results of this simulation are processed to produce regressions that connect diffuse reflectance of the seawater for different hydrooptical situations to the sun elevation angle.

INTRODUCTION

In papers by Haltrin [1], Gordon [2], Morel and Gentili [3] it was shown that the dependence of the diffuse reflection on the sun zenith angle is important and should be incorporated into remote sensing algorithms.

Two approaches to this problem are considered. The first approach is an analytical one and consists of approximate solution to the radiative transfer equation. As a result it gives Eqn. (20) for diffuse reflectance of the sea as a function of the inherent optical properties, sun elevation angle and the ratio of the direct illumination by the sun to the diffuse illumination by the sky. The second approach is a numerical Monte Carlo simulation. It is based on the modified by the author [4] approach by Kirk [5]. The results of this simulation are processed to produce a number of regressions that connect the diffuse reflectance of the seawater for different hydrooptical situations with the sun elevation angle. The results of both methods show that the corrections to the diffuse reflection of seawater due to the conditions of illumination can reach 40% at certain angular elevations and inherent optical properties of seawater.

THEORETICAL APPROACH

Let us consider a homogeneous sea illuminated by the skylight and the light of the sun elevated at h_s degrees. According to the Snellius law, the direct sunlight enters the sea at the angle $\cos^{-1}\mu_s$ from the vertical axis $0z$, directed from the sea surface to the sea bottom, where

$$\mu_s = \sqrt{1-\cos^2 h_s / n_w^2}\,, \tag{1}$$

here n_w is a refraction coefficient of seawater.

Let us start with the system of two flow equations for the downward E_d and upward E_u irradiances proposed in Ref. [1] and incorporated later into the elastic part of Ref. [6]:

$$\begin{aligned} &\left[\frac{d}{dz}+(2-\bar{\mu})(a+b_B)\right]E_d(z)-(2+\bar{\mu})\,b_B\,E_u(z)=f(z),\\ &-(2-\bar{\mu})\,b_B\,E_d(z)+\left[-\frac{d}{dz}+(2+\bar{\mu})(a+b_B)\right]E_u(z)=f(z). \end{aligned} \tag{2}$$

Here z is the depth coordinate, $\bar{\mu}$ is the average cosine over the irradiance angular distribution in the sea depth:

$$\bar{\mu}=\sqrt{\frac{1-x}{1+2x+\sqrt{x(4+5x)}}},\quad x=\frac{b_B}{a+b_B}\equiv\frac{B\omega_0}{1-\omega_0+B\omega_0}, \tag{3}$$

$b_B = b\,B$ is the backscattering coefficient, b is the scattering coefficient, $B=0.5\int_{\pi/2}^{\pi}p(\vartheta)\sin\vartheta\,d\vartheta$ is the probability of backscattering, $p(\vartheta)$ is the scattering phase function, ϑ is the scattering angle, x is the Gordon's parameter, $\omega_0=b/c$ is the single scattering albedo, $c=a+b$ is the attenuation coefficient, a is the absorption coefficient. The source functions in the right parts of Eqns. (2) are equal to:

$$f(z)=b_B\,E_S\exp\left(-\alpha\,z/\mu_s\right),\quad \alpha=a+2\,b_B, \tag{4}$$

where $\mu_s E_S$ is the sun irradiance just below the sea surface. In Eqns. (2) the direct sunlight is taken into account in the form of the source functions $f(z)$.

SOLUTIONS FOR IRRADIANCES

Solutions of Eqns. (2) with $f=0$ for the case of purely diffuse illumination of the sea surface are given in Refs. [1]. Let us find the solutions of these equations for the case of combined illumination, *i. e.* for the case when $f\neq 0$. Let us consider the irradiance of the skylight penetrated into the sea E_0 by the following boundary condition:

$$E_d(0)=E_0. \tag{5}$$

In addition let us accept that just below the sea surface the directed irradiance of the sun q times stronger than the irradiance of the sky, *i. e.*, $E_S = q\,E_0$. Let us also define the following quantities: let $E_u^D(z)$ be the portion of the diffuse light that is originated from the scattering of the skylight penetrated into the ocean, and let $E_u^S(z)$ be the portion of the diffuse light that is originated from the scattering of the sunlight penetrated into the sea.

Let us introduce the following definitions:

$$R_\infty = E_u^D(0)/E_d(0) \tag{6}$$

is the diffuse reflectance of the infinitely optically deep ocean illuminated by diffuse light; let

$$R_S = E_u^S(0)/(\mu_s E_S) \tag{7}$$

be the diffuse reflectance of the infinitely optically deep ocean illuminated by directed light of the sun; let

$$R_C = E_u(0)/[E_d(0)+\mu_s E_S], \qquad (8)$$

be the diffuse reflectance of the infinitely optically deep ocean illuminated by the combined light of sun and sky. Then the diffuse reflectance of the sea illuminated by the natural light is:

$$R_C = S_H\, R_\infty, \quad S_H = \frac{1+\mu_s\, q\, R_S/R_\infty}{1+\mu_s\, q}. \qquad (9)$$

Let us look for a solution of the system of Eqns. (2) in the form of the sum of general and partial solutions:

$$E_d(z) = A_d\, e^{-\alpha_\infty z} + C_d\, e^{-\alpha z/\mu_s}, \qquad (10)$$
$$E_u(z) = A_u\, e^{-\alpha_\infty z} + C_u\, e^{-\alpha z/\mu_s},$$

here $-\alpha_\infty = -a/\bar{\mu}$ is a negative eigenvalue of Eqns (2). Inserting Eqns. (10) into Eqns. (2) and applying boundary condition (5) we have following equations for irradiances:

$$E_d(z) = E_d^D(z)+E_d^S(z),\; E_u(z) = E_u^D(z)+E_u^S(z), \qquad (11)$$

$$E_d^D(z) = E_0\, e^{-\alpha_\infty z},\; E_u^D(z) = R_\infty E_0\, e^{-\alpha_\infty z}, \qquad (12)$$

$$E_d^S(z) = \mu_s q\, E_0 \frac{R_0\left(R_p+R_S\right)}{1-R_0\, R_\infty}\left(e^{-\alpha z/\mu_s} - e^{-\alpha_\infty z}\right) \qquad (13)$$

$$E_u^S(z) = \mu_s q\, E_0\left[R_S\, e^{-\alpha_\infty z} + \frac{R_S + R_p\, R_0\, R_\infty}{1-R_0\, R_\infty}\left(e^{-\alpha z/\mu_s} - e^{-\alpha_\infty z}\right)\right]$$

here (14)

$$R_\infty = \left(\frac{1-\bar{\mu}}{1-\bar{\mu}}\right)^2, \quad R_0 = \frac{2+\bar{\mu}}{2-\bar{\mu}} R_\infty, \qquad (15)$$

$$R_S = \frac{\bar{\mu}\, s(1+R_\infty)}{\mu_s[1+2\,\bar{\mu}^2(1+s)]+\bar{\mu}\,(1+2\,s)}, \qquad (16)$$

$$R_p = \frac{\bar{\mu}\, s(1+R_0^{-1})}{\mu_s - \bar{\mu}\,(1+2\,s)}, \quad s = \frac{b_B}{a} \equiv \frac{B\omega_0}{1-\omega_0}. \qquad (17)$$

DIFFUSE REFLECTANCE COEFFICIENTS

After some algebra we have the following equations for the diffuse reflection coefficients of the sea illuminated by the different kinds of light: 1) the diffuse reflection coefficient of the sea illuminated by the light of the sky:

$$R_\infty = \left(\frac{1-\bar{\mu}}{1-\bar{\mu}}\right)^2, \quad \bar{\mu} = \sqrt{\frac{1-x}{1+2x+\sqrt{x(4+5x)}}}, \qquad (18)$$

where $x = b_B/(a+b_B) \equiv B\omega_0/(1-\omega_0+B\omega_0)$;

2) the diffuse reflection coefficient of the sea illuminated by the direct sunlight:

$$R_S = \frac{(1-\bar{\mu})^2}{1+\mu_s\bar{\mu}\,(4-\bar{\mu}^2)}, \quad \mu_s = \sqrt{1-\cos^2 h_s/n_w^2}; \qquad (19)$$

and 3) the diffuse reflection coefficient of the sea illuminated by the combined light of the sky and the sun:

$$R_C = \frac{R_\infty + \mu_s\, q\, R_S}{1+\mu_s\, q}. \qquad (20)$$

The diffuse reflection coefficient R_C depends on the inherent

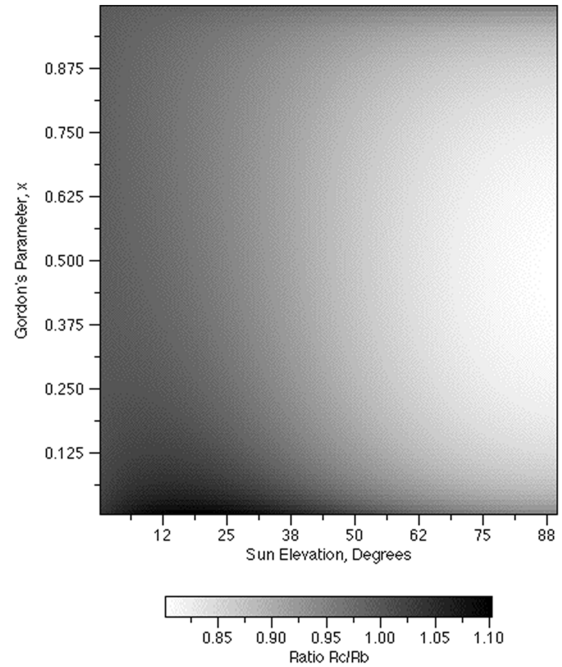

Fig. 1. Two-dimensional density plot of the ratio S_H as a function of the Gordon parameter x and the Sun elevation angle.

optical properties of the water b_B and a, the sun elevation angle h_s, and the parameter q that determines the ratio of the direct sunlight to the light of the sky. Correct evaluation of this parameter should involve solution of the radiative transfer problem in atmosphere [7] and estimation of the transmission through the air-sea interface. More simple approach was proposed by Jerlov [8]. According to Ref. [8], in the case of the clear sky the ratio q depends only on the sun elevation angle h_s. Our approximation of the data published in Ref. [8] gives the following formula:

$$q = 0.25(1+0.3\,h_s), \qquad (21)$$

here h_s is measured in degrees. Taking into account Eqns. (18)-(21), we have the following expression for the ratio of the diffuse reflectances $S_H = R_C/R_\infty$:

$$S_H = 1 - \frac{1-(1+\bar{\mu})^2 \Big/ \left[1+\bar{\mu}(4-\bar{\mu}^2)\sqrt{1-\cos^2 h_s/n_w^2}\right]}{1+4\Big/(1+0.3\,h_s)\sqrt{1-\cos^2 h_s/n_w^2}}. \qquad (22)$$

Figure 1 shows a two-dimensional density plot of the ratio S_H as a function of the Gordon parameter x and the sun elevation angle h_s. The difference between maximum and minimum values of this ratio lies in the range of 40%.

MONTE CARLO APPROACH

This approach is explained in detail in Ref. [4]. The main result of this approach related to the influence of sun elevation is the following regression equation:

$$r_\mu = R_S(90°)/R_S(\mu_s) = -c_0 + c_1\,\mu_s - c_2\,\mu_s^2, \qquad (23)$$

that relates the ratio of diffuse reflectance coefficients to the cosine of the sun penetration angle μ_s. The coefficients of these regressions are given in Table 1. The difference between the theoretical diffuse reflectance (20) and the numerically simulated values do not exceed 20%.

Table 1. Coefficients c_0, c_1, and c_2 for each of the fifteen Petzold phase functions [9, 10].

#	x	c_0	c_1	c_2	r^2
01	0.03444	0.288392	1.58743	0.29775	0.999
02	0.01408	0.565779	3.23327	1.66143	0.995
03	0.01322	0.576934	3.18952	1.60944	0.997
04	0.01936	0.967853	3.86569	1.89692	0.999
05	0.01566	1.024818	4.03269	2.00333	0.999
06	0.08194	0.852371	3.32589	1.46969	0.999
07	0.09064	0.810208	3.24149	1.42623	0.999
08	0.14786	0.477916	2.38229	0.90046	0.999
09	0.01139	-0.02428	2.01621	1.03560	0.987
10	0.06660	0.847081	3.28663	1.43518	0.999
11	0.01265	1.108715	4.21634	2.10450	0.998
12	0.08947	0.762611	3.03790	1.26988	0.999
13	0.03550	1.057443	3.92840	1.86609	0.999
14	0.02131	0.718718	3.41994	1.69663	0.999
15	0.01355	0.040812	2.19756	1.14971	0.990

CONCLUSIONS

This paper presents two approaches to the problem of angular dependence of the sea diffuse reflection coefficient. One of the approaches is analytical and it consists of the approximate solution to the radiative transfer equation. The result of this approach is Eqn. (20) for diffuse reflectance of the sea as a function of inherent optical properties, the sun elevation angle and the ratio of direct illumination by the sun to the diffuse illumination by the sky. The second approach is a numerical Monte Carlo simulation. The results of this simulation are regressions given by Eqn. (23) that connect ratios of the diffuse reflectances of the seawater at 90° and arbitrary degrees with the sun elevation angle.

The results of both methods show that the corrections to the diffuse reflection of seawater due to the illumination conditions can reach 40%. This effect is significant and should be taken into account in processing optical remote sensing data.

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